

Involute Gear Equations

Given:

P_n : Normal diametral pitch

Ψ : Helix angle at D

N: Number of teeth

ϕ_n : Normal pressure angle at D

Transverse diametral pitch

$$P_t = P_n \cdot \cos(\Psi)$$

Pitch diameter

$$D = \frac{N}{P_t}$$

Addendum, standard

$$a = \frac{1}{P_n}$$

Outside diameter

$$D_o = D + 2 \cdot a$$

Transverse pressure angle

$$\tan(\phi_t) = \frac{\tan(\phi_n)}{\cos(\Psi)}$$

Base diameter

$$D_b = D \cdot \cos(\phi_t)$$

Transverse pressure angle at any diameter

$$\cos(\phi_2) = \frac{D_b}{D_2}$$

Lead

$$L = \pi \cdot D \cdot \cot(\Psi)$$

$$L = \frac{\pi \cdot D}{\tan(\Psi)}$$

Helix angle

$$\cos(\Psi) = \frac{N}{D \cdot P_n}$$

$$\sin(\Psi) = \frac{\pi \cdot N}{P_n \cdot L}$$

Helix angle at any diameter

$$\tan(\Psi_2) = \frac{D_2 \cdot \tan(\Psi_1)}{D_1}$$

Normal circular pitch

$$p_n = \frac{\pi}{P_n}$$

Standard normal circular tooth thickness

$$t_n = \frac{p_n}{2}$$

Axial pitch

$$p_x = \frac{\pi}{P_n \cdot \sin(\Psi)}$$

$$p_x = \frac{p_n}{\sin(\Psi)}$$

$$p_x = \frac{L}{N}$$

Transverse circular pitch

$$p_t = \frac{\pi}{P_t}$$

$$p_t = \frac{p_n}{\cos(\Psi)}$$

Transverse circular pitch (pt2) at any diameter (D2)

$$p_{t2} = \frac{\pi \cdot D_2}{N}$$

Involute function of pressure angle

$$\text{inv}(\phi_t) = \tan(\phi_t) - \phi_t$$

Base helix angle

$$\cos(\Psi_b) = \frac{p_n}{p_b} = \frac{\cos(\Psi) \cdot \cos(\phi_n)}{\cos(\phi_t)}$$

$$\sin(\Psi_b) = \frac{p_n}{p_x} = \sin(\Psi) \cdot \cos(\phi_n)$$

$$\tan(\Psi_b) = \frac{p_b}{p_x} = \tan(\Psi) \cdot \cos(\phi_t)$$

Standard center distance

$$C = \frac{N_p + N_g}{2 \cdot P_n \cdot \cos(\Psi)}$$

Operating pitch diameter (Pinion) with nonstandard CD

$$D_{rp} = \frac{2 \cdot C \cdot N_p}{N_p + N_g}$$

Operating pressure angle with nonstandard CD

$$\cos(\phi_{rt}) = \frac{D_{bp} + D_{bg}}{2 \cdot C}$$

K Factor

Contact load factor for pitting resistance, lb/in²

$$K = \left[\frac{Wt}{dF} \right] \cdot \left[\frac{1}{C_g} \right]$$

Gear ratio factor, External gears

$$C_g = \frac{N_g}{N_g + N_p}$$

Gear ratio factor, Internal gears

$$C_g = \frac{N_g}{N_g - N_p}$$

Allowable contact load factor, lb/in²

$$K_{ac} = \left[\frac{I}{1.25 \cdot K_v \cdot C_g} \right] \cdot \left[\frac{sac \cdot ZN \cdot CH}{C_p} \right]^2$$

- I: Pitting resistance geometry factor
- K_v: Dynamic factor
- C_g: Gear ratio factor
- sac: Allowable contact stress number, lb/in²
- ZN: Stress cycle factor for pitting resistance
- CH: Hardness ratio factor for pitting resistance
- C_p: Elastic coefficient

Unit Load

Unit load for bending strength, lb/in²

$$UL = \frac{Wt \cdot Pnd}{F}$$

Allowable unit load for bending strength, lb/in²

$$Uat = sat \cdot YN \cdot \left[\frac{J}{1.25 \cdot \cos(\Psi_s) \cdot Kv \cdot KB} \right]$$

- sat: Allowable bending stress number, lb/in²
YN: Stress cycle factor for bending strength
J: Bending strength geometry factor
 Ψ_s : Helix angle at standard pitch diameter
Kv: Dynamic factor
KB: Rim thickness factor